FORMATION OF A "HANGING" COMPRESSION SHOCK IN THE FLOW PAST A BODY WITH A DISCONTINUITY IN ITS GENERATRIX

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The inevitability of the appearance of a "hanging" shock in the flow past bodies with a discontinuous generatrix (in the form of a sharp corner) and the properties of such shock wave in a plane flow of a perfect gas are established analytically. The case of a low-velocity oncoming stream, where entropy variations in the shock wave can be neglected, is considered ; use is made of the possibility of a transonic approximation of the shock polar line and its intersecting characteristics.

Ivanov [1] obtained a flow with a "hanging" compression shock in the region behind the front shock wave while computing the flow past a body with generatrix possessing a sharp corner. This effect had been previously observed in experiments but its cause remained obscure; Lighthill ([2], p. 366), for example, suggested that the shock could have been caused by the detachment and subsequent adhesion of the boundary layer in the neighborhood of the corner.

Let us consider the flow of a uniform supersonic stream past a body whose generatrix contains a sharp corner (the angle is convex at the corner of the generatrix). Without loss of generality we assume that the flow is symmetric, so that we need to investigate only the upper half-plane of the flow. The corner point (which we denote by A) separates the front segment OA of the contour from the rear segment AF (O is the leading edge of the profile in the case of an attached shock wave or the critical point in the case of a detached wave and F is its trailing edge). The segment OA (of finite length) is generally assumed to be curvilinear; the case of a straight segment OA, when the front shock wave is attached and the flow behind it supersonic, will be considered separately. The segment AF will be assumed to be essentially rectilinear. We denote the angle of its inclination to the axis of symmetry by β_0 (taken as positive in the counterclockwise direction). The length of the segment AF is infinite for $\beta_0 \geqslant 0$ and finite for $\beta_0 < 0$. If the segment AF is curvilinear, then β_0 is taken either as the angle of inclination of the profile at the point F (when AF is of finite length) or the angle of inclination of its asymptote (when AF is of infinite length). For $\beta_0 > 0$ we assume that β_0 is so small that a flow with an attached shock wave (behind which the flow is supersonic) exists near the wedge of vertex angle $2\beta_0$; we assume that such flow occurs at an infinite distance.

We denote, respectively, by β_1 and β_2, the angles of inclination to the axis of symmetry of the tangents to the segments OA and AF of the profile at point A ; owing to the profile convexity at the corner point, $\beta_1 > \beta_2$.

Definitions:

1) the term "shock wave" will be applied to the front shock wave which arises in front of the profile in a uniform supersonic flow. If the shock wave is detached, we assume that the region of subsonic velocities is bounded by the sonic line extending from the point A to the shock wave;

2) the term "secondary compression shock" will be applied to a compression shock in the region behind the front shock wave;

3) the term "hanging compression shock" will be applied to a secondary compression shock on both sides of which the flow is supersonic. By analogy with characteristics, we refer to it as a shock of the first (second) family, if its tangent is obtained by rotating the velocity vector by a positive (negative) acute angle. Hanging shocks belonging to different families do not link with each other smoothly.

The analysis is carried out by conformal mapping of the region behind the shock wave from the Cartesian system of coordinates xy (the x-axis coincides with the axis of symmetry) into the velocity hodograph plane $\eta\beta$ ($\eta = (k+1)^{1/3}(\lambda - 1)$, where λ is the velocity coefficient, β is the angle of inclination of the velocity vector to the axis of symmetry, k is the adiabatic exponent; the axes β and y are directed vertically upward; the axes λ and x are directed horizontally to the right. We denote points in the physical plane by capital letters and their images in the hodograph plane by lower-case letters.

First, let us consider the flow with an attached shock wave past a profile consisting of straight-line segments OA and AF for $\beta_0 = \beta_2 = 0$. In Fig. 1 OB is the straight segment of the shock wave; AB, AC, AD are straight segments of characteristics of the first family; BE is a characteristic of the second family.

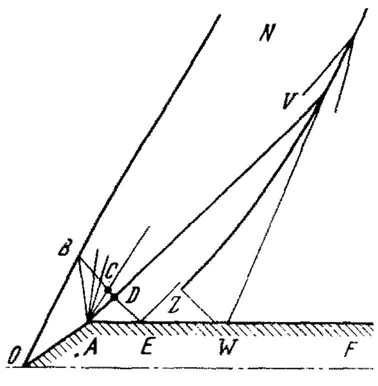

Fig. 1

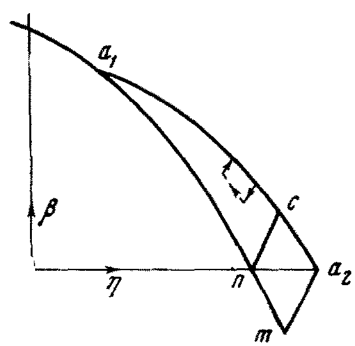

Fig. 2

Let us map the region behind the shock wave into the plane $\eta\beta$.

The image of the region OBEAO is the segment a_1a_2 of the second-family characteristic $\beta = C - {}^2/_3\eta^{3/2}$ (Fig. 2); the point a_1 with the ordinate β_1 lies on the shock polar $\beta = 2^{-1/2}(\eta_\infty + \eta)^{1/2}(\eta_\infty - \eta)$; the point a_2 lies on the axis η. Equations of the characteristics and of the shock polar are given in the transonic approximation.

L e m m a 1. A segment of the characteristic drawn from a point of the shock polar in the direction of increasing η lies outside the shock polar loop.

It is easy to verify that for $0 < \eta < \eta_\infty$ the angle of inclination of the characteristic of the axis η at the shock polar is smaller in absolute value than that of inclination of the shock polar. Now we need only recall that in the plane $\eta\beta$ the characteristics are invariant with respect to translation in the direction of the β-axis.

Lemma 1 implies that $\eta > \eta_\infty$ at the point a_2.

Thus, in the upper half-plane $\eta\beta$ there exists a point of intersection of the characteristic segment a_1a_2 and the first family characteristic nc drawn from the point n of the shock polar which represents the uniform flow in front of the shock wave.

Theorem 1. If the flow in the region behind the shock wave in a flow with an attached shock wave past a profile consisting of straight-line segments OA and AF is supersonic, then it is not continuous.

Let us consider in the physical plane the node of the first family characteristics emerging from the point A (called in the following "node A"). If the flow behind the shock wave is everywhere supersonic and continuous, then each characteristic of the node either intersects the shock wave or extends to infinity. Accordingly, in the hodograph plane each first family characteristic passing through the characteristic a_1a_2 must reach either the shock polar or the point n, which also lies on the shock polar and represents a uniform straight-line flow at infinite distance.

We thus have a contradiction, since the first family characteristics emerging in the $\eta\beta$-plane from points of the segment ca_2 cannot reach the shock polar. Hence, if the flow behind the shock wave is supersonic, a secondary compression shock must appear.

With suitable alteration in its formulation, Theorem 1 is also valid for the case of a detached shock wave; in accordance with the "law of monotonicity" of the velocity vector at the sonic line [3], the characteristic representing the corner point lies outside the shock polar (above it) in the plane $\eta\beta$.

The assumptions about the rectilinearity of the segments OA and AF are not essential. If the profile is curvilinear and $\beta_0 < 0$, we can prove our statement by specifying $\beta_2 < \beta_c$. If the segment is rectilinear, this condition becomes $\beta_0 < \beta_c$.

Let us suppose that one or more secondary shocks generated in the region behind the shock wave are hanging shocks of the first family. Let us determine the basic properties of these compression shocks, which we shall henceforth refer to simply as shocks. In the plane $\eta\beta$ the images of points along the shock (in front and behind the shock) lie on the shock polar segment $\beta_2 = \beta_1 + 2^{-1/2}(\eta_1 + \eta_2)^{-1/2}(\eta_1 - \eta_2), \qquad \beta_2 \geqslant \beta_1$

Lemma 2. The image of the vertex of the profile convex angle in the $\eta\beta$-plane is a segment of a characteristic which is devoid of cusps. The image of the vertex neighborhood covered by the characteristics of the node (lying at this vertex) in the plane $\eta\beta$ is one-sheeted and lies on one side of the characteristic – the image of the corner vertex.

That the corner point is represented in the hodograph plane by a characteristic is shown in [4]. This implies that the node of characteristics of another family lies at this point.

As we know, the image of the physical plane xy in the plane $\varphi\psi$ for $0<\lambda<(k+1)^{1/2}(k-1)^{-1/2}$ is a one-to-one map (φ is the velocity potential and ψ is the stream function) [4]. This means that in the $\varphi\psi$-plane the corner point is represented by a point. The angle of inclination γ of the characteristics in the plane $\varphi\psi$ to the line $\psi = \text{const}$ is calculated by the formula

$$\operatorname{ctg}\gamma = \frac{d\varphi}{d\psi} = \frac{h_2 ds_1}{h_1 ds_2} = \frac{\operatorname{ctg}\mu}{\varepsilon(\lambda)} = (M^2-1)^{1/2}\left(1+\frac{k-1}{2}M^2\right)^{1/(k-1)} \qquad (1)$$
$$ds_1 = h_1 d\varphi, \quad ds_2 = h_2 d\psi$$

Here h_1, h_2 are the Lamé coefficients, M is the Mach number, and μ is the Mach angle.

In the plane $\varphi\psi$ the characteristics emanating from the corner point lie at the dicritical node in the order of decreasing γ, i.e. in the order of increasing velocity. Hence, in the presence of a cusp in the characteristic representing the corner point in the plane $\eta\beta$, the plane $\varphi\psi$ (hence, also, in the physical plane) would contain a fold, which is impossible.

Let us consider in the plane $\eta\beta$ the image of the neighborhood G of the corner point covered by the node characteristics. Let us suppose that the characteristic segment a_1a_2 representing the corner point, contains a point such that to the left of it G lies to one side of a_1a_2, and to the right of it, it lies to the other side of this segment. In this case there must be a fold in the map in the plane $\eta\beta$; as we know [4], the edge of the fold (the branch line, where the Jacobian $\partial(\eta, \beta) / \partial(x, y)$ changes sign) in a potential flow is a characteristic. This characteristic and the image of the corner point belong to different families; hence, the branch line is a node characteristic. It is also clear that along the node characteristics arbitrarily close to this characteristic and lying to different sides of it, the velocity distributions would be different (in the plane $\eta\beta$ they lie to different sides of the characteristic a_1a_2). This is impossible, since along a characteristic the flow is continuous.

It is convenient at this juncture to formulate separately the known property of relative disposition of characteristics and the compression shock (see e.g. [2]).

Lemma 3. The angle of inclination of a compression shock to the velocity vector ahead of the shock (behind the shock) is larger (smaller) than the Mach angle at the corresponding point.

We apply the term "origin" ("end") of a hanging shock to that point of the shock where the value of ψ at the shock is smallest (largest) when ψ is measured from the profile.

Theorem 2. The shock origin cannot lie on inner characteristics of node A.

A shock cannot emerge from the point A, if its intensity at this point is zero and if it is tangent to one of the inner node characteristics at this point, since this would constitute a violation of the analyticity of the Prandtl-Meyer solution.

It is also impossible for a shock to emerge from the point A, if its intensity there is not zero, since in this case the shock line would belong to the dicritical node of characteristics at the point A, which would imply multivalence in the physical plane.

Now let us suppose that the shock originates some distance from the point A in the region covered by node characteristics. Lemma 3 implies that the characteristics which emerge from the point A travel in the reverse direction after intersecting the shock and therefore arrive at the point A once again. This cannot happen, since on connecting in the plane $\eta\beta$ the characteristics which emerge from a_1a_2 by a segment of the shock polar corresponding to the first-family shock (the groken line in Fig. 2), we find that the map of the neighborhood of the point A is multivalent, which contradicts Lemma 2.

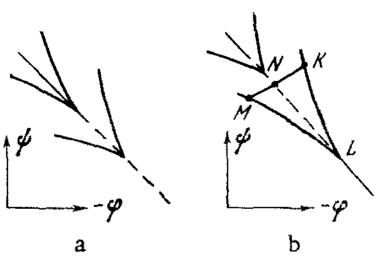

Fig. 3

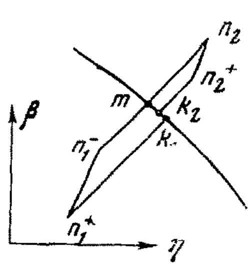

Fig. 4

Let us consider the particular case of flow past a profile with straight segments OA and AF when the flow behind the attached shock wave is supersonic. We denote by H the region bounded by the second-family characteristic ED and the first-family characteristics which pass through the points E and D in the direction away from the profile (Fig.1). Since the region H is adjacent to the triangle ADE in which the flow is uniform and rectilinear, it follows that the flow in this region is a simple wave (the second-family characteristics are straight). The characteristic ADN is a boundary of the region H.

Theorem 3. In the case where there exists a region H, the origin of the shock lies neither in H nor on the characteristic ADN.

Let us suppose that the opposite statement holds. The image in the plane $\eta\beta$ of the shock line (behind the shock) in some neighborhood of its origin is a first-family characteristic which passes through the point a_2.

Constructing the segment of the shock polar corresponding to a first-family shock through any point of this characteristic, we find that the image of the shock line (in front of the shock) lies outside the region covered by the first-family characteristics which emerge from the segment a_1a_2. This contradicts Lemma 3, since the first-family characteristics constructed from the shock (in some neighborhood of its origin) in the region in front of the shock either belong to node A or lie in the region H.

If a decrease of the velocity potential φ corresponds to some displacement in the physical plane, we say that this displacement is "downstream".

Lemma 4. A hanging shock lies downstream from its terminal point (its origin or end). (*)

Without loss of generality, we confine ourselves in proving this lemma to the second-family hanging shock shown in Fig. 3 in the coordinates $\varphi\psi$ (the solid curves are characteristics, the broken lines are shocks). We propose to show that the case appearing as Fig. 3b is impossible.

Let us suppose that the opposite statement holds. In accordance with Lemma 3, two second-family characteristic rays which lie on opposite sides of the shock and are tangent to each other, emerge upstream of point L (the origin of the shock). Let M be a point on the characteristic ray in the region in front of the shock chosen sufficiently close to the point L; MK is a first-family characteristic; K is a point on the second-family characteristic ray LK (behind the shock); N (N^-, N^+) is the point of intersection of the characteristic MK with the shock ahead and behind the shock.

Since the rays ML and KL are tangent at the point L we have $|\gamma_K| > |\gamma_M|$.

In fact, if the characteristics ML and KL have opposite convexities (one towards the other), then near the point L we have

$$|\gamma_K| > |\gamma_L| > |\gamma_M|$$

Now let the characteristics near L have their convexities similarly directed. Let us take the equations of characteristics in the form $y = y(x)$ and $y_0 = y_0(x_0)$; the angles γ are then equal at the points (y, x) and (y_0, x_0). This means that

$$dy / dx = dy_0 / dx_0, \qquad y - y_0 = k(x_0)(x - x_0)$$

*) In [5], p. 553 (Fig. 109a) the authors express the supposition that an "arriving" hanging shock cannot degenerate at the sonic line or near it; the proof of Lemma 4 subsumes also this case.

Here $k(x_0)$ is the slope of the straight lines connecting points with equal γ on the characteristics KL and ML. Differentiating the second equation with respect to x_0 and using the first equation, we obtain

$$\frac{dk}{d\ln s}+k=\frac{dy_0}{dx_0}, \qquad s=x-x_0$$

Integration of this equation yields

$$k=\frac{1}{s}\left(C+\int\frac{dy_0}{dx_0}ds\right)=\frac{1}{s}\left[C+\left(\frac{dy_0}{dx_0}\right)_L s+\left[\frac{d}{ds}\left(\frac{dy_0}{dx_0}\right)\right]_L\frac{s^2}{2}+\ldots\right]$$

The constant C appearing in this expression must be equated to zero, since otherwise k would become infinite as $s\to 0$ in any coordinate system x, y, which is impossible.

Thus, we have $k=dy_0/dx_0$ at the point L. If the velocity is supersonic at the point L, then the direction of the first-family characteristics at the points M, K near L differs from that of lineas $\gamma=\text{const}$. From this we readily infer that $|\gamma_K|>|\gamma_M|$, so that (1) implies that $\eta_K<\eta_M$. If the velocity at the point L is sonic, the same result follows from the analysis of the formula for k with the sign of the derivative

$$d(\cdot)/ds\,(dy_0/dx_0)$$

in the neighborhood of the point L taken into account.

Consideration of the mapping of the neighborhood of the point L into the plane $\eta\beta$ with the aid of Lemma 1 yields the opposite inequality $\eta_K>\eta_M$. Figure 4 shows the two possible positions of the point n^- (n_1^- and n_2^-) relative to the point m. Here $n_1\ n_2$, $n_1^+k_1$, $n_2^+k_2$ are first-family characteristics; $n_1^-n_1^+$, $n_2^-n_2^+$ are segments of the second-family shock polars.

Corollary 1. The end of a first-family shock (where it degenerates into a characteristic) lies infinitely far from the profile.

Corollary 2. If a first-family shock intersects either the shock wave or another shock, its intensity is not equal to zero at the point of intersection.

Lemma 5. At the shock wave $J=\partial(\eta,\beta)/\partial(x,y)\leqslant 0$.

Since $\eta<\eta_\infty$ at points of the shock wave, for small η_∞ the flow in the neighborhood of the shock wave is described by the transonic flow equations

$$(k+1)^{1/3}\eta\eta_\varphi-\beta_\psi=0, \qquad (k+1)^{1/3}\beta_\varphi-\eta_\psi=0 \tag{2}$$

Adding to the above equations the derivatives in the direction of the shock wave simplified for the transonic velocity range,

$$\beta_\varphi\delta+\beta_\psi=\frac{d\beta}{ds}=\frac{d\beta}{d\delta}\varkappa, \qquad \eta_\varphi\delta+\eta_\psi=\frac{d\eta}{ds}=\frac{d\eta}{d\delta}\varkappa$$

we obtain a system for finding the derivatives $\eta_\varphi,\eta_\psi,\beta_\varphi,\beta_\psi$; these derivatives are proportional to $\varkappa$, as is the right side of the system.

Here δ is the angle of inclination of the shock wave to the vertical axis, $d(\cdot)/ds$ is the derivative in the direction of the shock wave; the derivatives $d\beta/d\delta$ and $d\eta/d\delta$ are computed from the relations at the shock wave in the transonic approximation,

$$\beta=\frac{2\delta}{(k+1)^{1/3}}\left[\eta_\infty-\frac{\delta^2}{(k+1)^{2/3}}\right], \quad \eta=\frac{2\delta^2}{(k+1)^{2/3}}-\eta_\infty, \quad \delta=(k+1)^{1/3}\sqrt{\frac{\eta_\infty+\eta}{2}} \tag{3}$$

The Jacobian at the shock wave satisfies the relation

$$J=\frac{\partial(\eta,\beta)}{\partial(x,y)}=\frac{\partial(\eta,\beta)}{\partial(\varphi,\psi)}=\frac{2}{(k+1)}(\eta-\eta_\infty)\varkappa^2\leqslant 0$$

As we know [4], in the subsonic range $J \leqslant 0$.

Corollary. If a characteristic is a branch line, its reflection from the shock wave is also a branch line (a characteristic of another family).

Lemma 6. If a certain segment of a characteristic is a branch line, then the entire characteristic is a branch line.

Let us suppose that a certain point inside the flow region bounds a segment of a characteristic which is a branch line. Since a branch line cannot terminate inside a flow region, this point also bounds the branch line (a segment of a characteristic belonging to another family). Thus, the neighborhood of this point in the physical plane is divided by four characteristic rays into four sectors, in three of which J has the same sign, and the opposite sign in the fourth. This is impossible, since the domains with differing signs of J could then be mapped univalently onto the same sector formed in the plane $\eta\beta$ by two characteristic rays belonging to different families.

Let us denote by Q the region bounded by the shock wave, by the segment OA of the profile, by a hanging shock, and by the last characteristic of node A (Fig. 1). In accordance with Theorems 2 and 3 the origin of a shock lies either on this characteristic or on a first-family characteristic lying downstream of it. Considering the mapping of the domain Q into the hodograph plane, we find that the flow in the domain Q does not depend on the shape of the segment AF of the profile. This property was noted in [1]. Let us take, for example, the case of a straight segment OA when the flow behind the attached shock wave is supersonic. The image of the region Q lies in the region T bounded by the shock polar a_1n, by the second-family characteristic a_1a_2 (at the point a_2 $\beta = \beta_0$), and by the first- and second-family characteristics a_2m and nm (Fig. 2).

To determine the stream function ψ in the region T, we specify at the characteristic a_1a_2 and at the shock polar the boundary conditions

$$\psi = f(\eta), \quad \psi_\eta - \psi_\beta \left(\frac{\eta_\infty + \eta}{2}\right)^{1/3} \frac{7\eta_\infty + \eta}{5\eta_\infty + 3\eta} = 0$$

The function $f(\eta)$ is defined as the value of the stream function at the second-family characteristic BD (Fig. 1).

In the case of a detached shock wave the region T contains the minimum region of influence of the segment OA; the boundary of the supersonic-velocity subregion of the region T consists of the indicated parts plus the segment of the line $\eta = 0$ which connects the shock polar and the characteristic a_1a_2.

Theorem 4. The mapping of the domain Q into the plane $\eta\beta$ is one-sheeted, and $J \leqslant 0$.

As we know, $J \leqslant 0$ for $\eta \leqslant 0$, so that it is sufficient to prove the univalent character of the mapping of the supersonic subregion of T.

Lemma 2 implies that the first-family characteristics (they belong to the node A) cannot be branch lines. Lemmas 5 and 6 also imply that second-family characteristics which emerge from the shock wave cannot be branch lines, since otherwise each of them would be a reflection from the shock wave of a branch line which is a first-family characteristic emerging from the node A. In the case of a detached shock wave the boundary of the subregion of T contains the sonic line; applying instead of Lemma 5 the "law of monotonicity" [3] together with Lemma 2, we find that characteristics emerging from the sonic line also cannot be branch lines. Applying Lemma 5, we find that $J \leqslant 0$ in the region T.

Theorem 4 implies the following corollary.

The segment of the shock wave which bounds the region Q has its convex side directed toward the oncoming stream.

Lemma 7. Along the sraight segment of the profile $J \leqslant 0$ for $\eta < 0$ and $J \geqslant 0$ for $\eta > 0$.

Using (2), we transform the expressions for J, and obtain

$$J = (k+1)^{1/3}\eta\eta_\varphi^2 \quad \text{for} \quad \partial\beta / \partial\varphi = 0$$

Corollary. If a characteristic is a branch line, then its reflection from a rectilinear profile (a characteristic of a different family) is also a branch line.

Let us denote by Z the origin of the shock bounding the region Q, and by V the point of intersection of the last characteristic of node A with the shock (if the shock begins elsewhere than on this characteristic). Let us extend the secod-family characteristic ZW until it intersects the profile at the point W. Let P be either the region bounded by the characteristics AZ, ZW and by the profile segment AW (if the point Z lies on the last node characteristic) or the region bounded by the characteristic AV, the shock VZ, the characteristic ZW, and the profile segment AW.

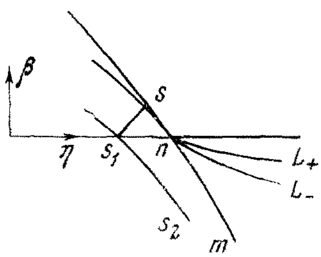

Fig. 5

From Theorem 4 and Lemmas 6 and 7 we have:

Theorem 5. In the flow past a profile with a rectilinear segment AF the last characteristic of node A is a branch line. (If a region H exists, then J has different signs on either side of this region). The mapping of the domain P onto the plane $\eta\beta$ is one-sheeted, and $J \geqslant 0$. The velocity decreases monotonically as we travel away from the point A along the segment AF.

Theorem 6. In the flow past a profile with a straight segment AF for $\beta_0 \leqslant 0$ a shock does not intersect the shock wave; moreover, there exist no other hanging first-family shocks. (*)

Let us denote the $\eta\beta$-image of the shock line bounding the region Q in the region ahead of the shock by L_- and in the region behind the shock by L_+. The line L_- lies in the region T. Since the corresponding points of the lines L_- and L_+ are jointed by first-family segments of the shock polars, we can apply Lemma 1 to find that the line L_+ lies in the plane $\eta\beta$ in the region situated above the second-family characteristic mn (we are referring to the entire characteristic).

Let us suppose that the shock which bounds Q intersects the shock wave; we denote the point of intersection by S. From the point S we extend a first-family characteristic SS_1 to the profile (Lemma 2 implies that the characteristic SS_1 does not intersect other first-family shocks, even if they exist). From the points S_1 we draw the second-family characteristic S_1S_2 to its intersection with either the shock or the shock wave at the point S_2.

By hypothesis the point S lies at a finite distance from the profile, and the velocity

*) This statement does not follow from the theory of flow at infinite distances from a profile [5]. As the present author was informed by O. S. Ryzhov, this theory admits of the existence of shock waves between the bow and tail shock waves, but with a more rapid law of intensity decrease.

at the point S (behind the shock) is lower than the velocity of the oncoming stream. Hence, the point s, which is the $\eta\beta$-image of S, does not coincide with the point n. In other words, $\beta_s > \beta_n$.

Having constructed the characteristics ss_1 and s_1s_2, we find that the point s_2 lies in the plane $\eta\beta$ under the characteristic mn (Fig. 5). This means that the point s_2 does not belong to the line L_+. Moreover, Lemma 1 implies that the point s_2 cannot lie on the shock polar. Thus, our assumption that a shock can intersect the shock wave has led to a contradiction.

To prove the uniqueness of the shock we merely need to show, in accordance with Lemma 2, that the region behind the shock which is covered by first-family characteristics (we denote this region by R) is not bounded downstream by any first-family characteristic.

Let us suppose that the opposite statement holds. Then there exists a "last" first-family characteristic α which "intersects" the shock at an infinite distance. We shall show that the image of this characteristic is the point n. In fact, the $\eta\beta$-image of the domain R and of the characteristic α lies above the characteristic mn. This means that on the profile segment which belongs to the region R we have $\eta \geqslant \eta_n$ (we note, recalling Lemma 7, that the same estimate holds on the line L_-, in some neighborhood of the point n). If the image of the characteristic lies above mn, then some neighborhood of its point of intersection with the profile lies above the straight line $\beta = 0$. However, this contradicts Lemma 7.

We therefore conclude that, if the characteristic α exists, the flow along this characteristic must be uniform and rectilinear. This in turn implies that in some subregion of R, which is bounded by a shock, the flow is a simple wave (the first-family characteristics are straight in this case). Hence, some segment of the line L_+ in the neighborhood of the point n lies on the characteristic mn. However, the corresponding segment of the line L_- would (by Lemma 1) lie below the characteristic mn, which is impossible.

Theorem 7. The following estimates are valid:

1) on the segment of the line L_- bounding the image of the region Q

$$d\eta / ds < 0, \qquad |d\beta / d\eta| \leqslant \sqrt{\eta}$$

2) on the segment of the line L_- bounding the image of the region P

$$|d\beta / d\eta| \geqslant \sqrt{\eta}$$

3) in flow past a profile with a straight segment AF for $\beta_0 \leqslant 0$ at the line L_+ in the neighborhood of the point n $\quad d\eta / ds < 0$

4) if a hanging shock originates at the last characteristic of node A, then in flow past a profile with a straight segment AF on the line L_+ in the neighborhood of the origin of the shock $\quad d\eta / ds < 0$

Here s is the length of the shock arc measured from the profile.

Estimates (1), (2) and (4) were derived from Theorems 4 and 5 and from Lemma 6 with the order in which the shock is intersected by characteristics in the physical plane taken into account. Estimate (3) follows from the condition obtained in proving Theorem 6, whereby $\eta \geqslant \eta_n$ at the line L_+ in the neighborhood of the point n.

Theorem 8. In the flow past a profile with a straight segment AF for $\beta_0 \lessgtr 0$ the shock at an infinite distance has its convex side towards the region behind the shock. (*)

Differentiating the second expression of (3) with respect to s, we obtain

$$\frac{d\eta_+}{ds}+\frac{d\eta_-}{ds}=\frac{4\delta}{(k+1)^{2/3}}\frac{d\delta}{ds}$$

Making use of Estimates (1) and (3) of Theorem 7, we arrive at the estimate $d\delta/ds \leqslant 0$.

In flow past a rhombic profile (for $\beta_0 < 0$), a "tail" shock emerges from the point F. If the dependence of the flow on the parameter β_0 is continuous, then at least for sufficiently small values of $|\beta_0|$ an inner shock arises in addition to the tail shock. In fact, as $\beta_0 \to 0$ the point F moves into infinity, and the tail shock vanishes, so that the shock whose existence was proved for $\beta_0 = 0$ is not a tail shock.

Theorem 6 with allowance for Lemma 3 implies the following.

Corollary. If an interior shock exists in flow past a profile with a straight segment AF and $\beta_0 < 0$, then the tail and interior shocks intersect, forming a V-shaped shock.

Let us consider the flow past a profile with a straight segment AF for $\beta_0 > 0$. A hanging shock need not arise for flow past such a profile (the condition of appearance of a shock was stated above).

Theorem 9. If a shock exists in the flow past a profile with a straight segment AF for $\beta_0 > 0$, then this shock intersects the shock wave. At the point of intersection at the shock wave we have $\beta < \beta_0$ behind the shock. The shock wave at the segment lying downstream from the point of intersection with the shock (or downstream from the point of intersection with the last characteristic of node A if a shock does not exist) consists of an infinite number of segments with pairwise-different signs of curvature of the shock wave; the oscillations of the angle of inclination decay with distance from the profile (the property of oscillation of the angle of inclination of the shock wave is compatible with the theory of propagation of perturbations in flow past a wedge-shaped profile [6]).

Let us suppose that a shock does not intersect the shock wave. Since the flow at an infinite distance from the point A becomes uniform and rectilinear, the shock degenerates at infinity into a first-family characteristic. A contradiction arises, since in uniform flow near a wedge the characteristic travels from the wedge towards the shock wave.

The intensity of a shock at the point S where it intersects the shock wave differs from zero (Corollary 2 of Lemma 4). A rarefaction wave then emerges from the point S, since the shock polar constructed from some point of a different shock polar does not intersect it (lies within it; this property of shock polars in the transonic approximation is noted in [7], in the remark on p. 182). It is easy to show that the rarefaction wave is represented by the node of second-family characteristics.

Just as we showed the impossibility of intersection of a shock and shock wave in Theorem 6, we can prove that $\beta < \beta_0$ behind the shock at the point S on the shock wave.

It is easy to show that the shock wave on the segment downstream from the point S is not straight; to this end we need merely extend the second-family characteristics of the rarefaction node to the segment AF and then draw their reflections (first-family characteristics) to the shock wave.

Let us take an arbitrary point on the shock polar downstream from the point S (if

*) The Corollary of Theorem 4 and Theorem 8 are compatible with the theory of flow at infinite distances from a profile [5].

there are several shocks, then we take the point of intersection with the last shock as our S). From this point we first construct the second-family characteristic to the profile, then the second-family characteristic to the shock wave, etc. The images of these characteristics in the plane $\eta\beta$ form a broken "helix" consisting of segments of the corresponding characteristics; applying Lemma 1, we find that this helix "winds around" the point of intersection of the shock polar with the straight line $\beta = \beta_0$. Thus, the shock wave consists of successive segments with differing signs of curvature; the "amplitude" of the oscillations of the angle of inclination of the shock wave decreases as we move along the shock wave in the direction away from the profile.

This property can be proved in similar fashion in the case where there is no compression shock.

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